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Last-Minute Bidding in Sequential Auctions with Unobserved, Stochastic Entry

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Abstract Using a dataset of calculator auctions on eBay, we first show that last-minute bidding is not merely the result of bidders' going to the next-to-close auction. Instead, bidding is concentrated at the end of the period in which the auction is the next to close, suggesting the existence of strategic last-minute bidding. Then, we model repeated, ascending price auctions for homogeneous goods with unobserved, stochastic entry. We show that the dynamic game has a pure-strategy symmetric equilibrium in which entrants always reveal themselves by bidding in the auction in which they arrive, and bidding occurs at the last minute.

Keywords eBay · Last-minute bidding · Sequential auctions

JEL Classification D44 · D8

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1 Introduction

In the United States, eBay has become the dominant player in the online auction market, with sales of over $8 billion in 2007 (data from http://forbes.com). An important feature of eBay is that auctions for identical products are held repeatedly, with many auctions that overlap and close one after another.[1] The availability of auctions that close at a later date allows bidders who are outbid in one auction to bid again in another auction. The objective of this paper is to explore how the option to bid again conditional on losing can affect bids, the timing of bids, and prices in repeated auctions of relatively homogenous goods.

Last-minute bidding (or "sniping") in eBay auctions is well documented.[2] However, we introduce a new and informative approach of studying bid submission times: we order the auctions consecutively according to their closing times and focus on the intervals in which each auction is next to close. Short intervals might make some buyers who bid in whichever auction ends next appear like strategic last-minute bidders. Our approach allows us to distinguish between these regular and last-minute bidders. We show that last-minute bidding is not merely the result of bidders' going to the next-to-close auction. Instead, bidding is concentrated at the end of the interval in which the auction is the next to close, which suggests the existence of strategic last-minute bidding.

In our theory section, we consider a simple model in which single units of a homogeneous good are sold in an infinite sequence of continuous-time, ascending-price auctions. Each auction lasts for one period, with a random number of new bidders who arrive each period (the "entrants"). Buyers have unit demands. Because more than one bidder may arrive during each period, there is also a stock of incumbent bidders who are waiting to obtain a unit; incumbents are bidders who entered an earlier auction and who have not yet won an auction. Each bidder has the same valuation. Thus, bidders are uncertain about the number of bidders but not about their valuations, and they use the outcomes of past auctions to predict the level of competition in the current and future auctions.

We derive a symmetric equilibrium in pure strategies in which entrants always bid in the auction during which they arrive. The equilibrium exhibits some striking features. First, bidders bid less than their true value. The markdown factor depends upon the number of competitors that bidders expect to face in subsequent auctions if they lose in the current auction. Second, entrants are more likely to win. They have an informational advantage over incumbent losers because the entrants know that they are competitors. Consequently, they expect higher levels of competition than do incumbents; and, as a result, they bid higher than incumbents. This selection effect arises from differences in information and not differences in valuations. Third, entrants bid at the last minute.

[1] For example, in our dataset of auctions of Texas Instruments TI-83 Graphing Calculators, on average an auction closes every 33 min, and the elapsed time between auctions is much smaller during peak hours of the day when more users are actively bidding. The median time between closings is 14 min.

[2] For a detailed summary see Hasker and Sickles (2010).

The organization of the rest of the paper is as follows: the next section describes eBay and some related literature. We present the data in Sect. 3 and the model in Sect. 4. Section 5 is the conclusion.

2 Background and Related Literature

eBay allocates items through auctions with proxy bidding, a mechanism that bids on behalf of the bidder up to a maximum bid specified by him. At the end of the auction, the bidder with the highest maximum bid wins the item at a price that is equal to the second-highest maximum bid submitted. The auction thus resembles an ascending second-price auction. The seller sets the auction's starting bid, which acts like a posted reserve price since it is observed by all potential buyers. eBay also allows sellers to set a secret reserve price that is not observable to the bidders. Potential buyers are informed by eBay via email and a posting on the website whether the reserve price is met or not. The seller selects the length of the auction: 1, 3, 5, 7, or 10 days.[3]

By choosing the starting date and length of the auction, the seller determines the precise day and time that it closes. There is no extension of auction length at the end of eBay auctions. This "hard-close" rule gives bidders an opportunity to delay their bid submissions and place their bids in the last minutes of the auctions. In a second-price setting with independent private values, Vickrey (1961) shows that bidders have a weakly dominant strategy to bid their valuations at any time during the auction. In practice, though, bidders on eBay tend to wait until the very end of the auction to submit their maximum bids.

Most of the theoretical research on eBay auctions ignores the multi-auction structure of eBay and focuses on the single unit auction. Roth and Ockenfels (2002) and Ockenfels and Roth (2006) argue that when buyers all submit maximum bids at the end of the auction, the resulting spike in network activity decreases the probability that an individual maximum bid gets recorded. Under certain conditions, buyers prefer to submit their maximum bids in the final seconds of the auction, in the hope that competitors' maximum bids will not be recorded. If the potential gain from reduced competition outweighs the risk of not getting in a maximum bid, then last-minute bidding is an equilibrium.

Bajari and Hortacsu (2003), Ockenfels and Roth (2006) and Rasmusen (2006) show that last-minute bidding can arise in a single-unit auction if valuations have a common component. In this environment, an early bid might reveal positive information about the bidder's estimate of the object's value. That signal would cause other bidders to bid more aggressively and raise the final price paid by the winner. One interpretation of our result is that the continuation value for losers introduces a common component to the valuations of the bidders.

Barbaro and Bracht (2006) suggest that last-minute bidding is a best response to dishonest actions—"shill bidding" and "squeezing"—by the sellers or sellers' accomplices. A shill bid is a high maximum bid that a seller submits in her own auction under

[3] When the data that we analyze in this paper were collected, one-day auctions were not an option for the sellers in eBay.

a false name. She thus can learn the value of the highest legitimate maximum bid. Then the seller cancels the shill bid and replaces it with another just below the highest legitimate maximum bid. In that way, any potential gain for the buyer is squeezed from him by the seller. Waiting until the very end of the auction to submit a maximum bid is a way to prevent shill bidding.

The main difficulty in studying eBay auctions as a sequence of single-unit auctions is that losing bids are observable. Since rivals can exploit this information in subsequent auctions, bidders have an incentive to "hide" their types, which often leads to non-existence of equilibria in monotone bid strategies (see Milgrom and Weber 2000). Wang (2006) gets around the problem by focusing on a model with only two goods. In the second auction, bidding one's valuation is a weakly dominant strategy, and so information about opponents' type does not affect bidding strategies. He shows that last-minute bidding in the first auction is the outcome of the unique symmetric sequential equilibrium in undominated, monotone strategies.

Zeithammer (2006) studies an infinite-horizon model of sealed-bid, private-value auctions when future auctions arrive randomly. His model is similar to ours in several respects, but he assumes that bidders have no memory and therefore cannot learn from the outcomes of past auctions. He does provide evidence that bidders lower their bids in response to higher arrival rates of close substitutes.[4]

Empirical studies of bidding in eBay auctions that take a structural approach also focus on the single-unit auction. Lewis (2011) and Bajari and Hortacsu (2003) argue that the prevalence of late bidding justifies modeling eBay auctions as a second-price sealed-bid, common-value auction. Gonzalez et al. (2009), Canals-Cerda and Pearcy (2006) and Ackerberg et al. (2006) adopt the independent private values model.[5] The first study assumes that entry is exogenous and that the number of potential bidders is constant across auctions. The latter two studies assume that the number of bidders is the stochastic outcome of a Poisson process. Bidders are assumed to bid upon arrival if their valuations exceed the standing high bid.[6]

The sale price in all three studies is determined by the second-highest valuation among the set of potential bidders, which plays a key role in identifying the underlying distribution of valuations. Consequently, all three studies assume that the bidder's continuation value conditional on losing is zero or, more generally, a constant that does not depend upon the level of competition (and therefore on auction outcomes). Our results suggest that this assumption is quite strong and unlikely to hold in thin markets.

Moreover, a qualitative difference between our last-minute bidding and other explanations lies in the different behavior of incumbents versus entrants: we expect to see

4 Other papers that examine sequential auctions include Ashenfelter (1989), Gale and Hausch (1994), Jeitschko (1999), Katzman (1999), McAfee and Vincent (1993), Milgrom and Weber (2000) and Pitchik and Schotter (1988). These papers either study the two-good case in which bidders have a dominant strategy in the second auction or assume that losing bids are not observable.

5 Interestingly, Canals-Cerda and Pearcy (2006) find that, in their dataset of paintings, last-minute bidding is not nearly as prevalent as in other datasets. The reason may be that the paintings are heterogeneous, so that losers are less likely to bid against each other in subsequent auctions.

6 However, they do not assume that bidders bid their valuations upon arrival, so that they can account for bidders that bid multiple times in an auction.

last-minute bidding more from entrants. As shown in the next section, we observe that the last-minute bidding activity is considerably higher for entrants than for incumbents.

3 Data and Analysis

In this section we establish three stylized facts that motivate our modeling assumptions: last-minute bidding occurs, buyers have unit demand, and losers in an auction bid again in future auctions with positive probability. We also demonstrate three features that are consistent with the predictions of our model: new entrants are both more likely to bid at the last minute and more likely to win the auction than are incumbents, and item prices increase with the number of losing bidders from recent previous auctions.

3.1 Data

Our dataset consists of information that was collected on Texas Instruments TI-83 Graphing Calculator auctions on eBay. The advantages of studying the TI-83 are a high volume of auctions and the relatively homogeneous nature of the product. The data cover every auction between June 15, 2003, and July 30, 2003.

Private auctions, in which information about the bidders is not available, and so-called "Dutch auctions" of multiple units are excluded from our dataset. We also exclude "Buy-It-Now" auctions, in which a buyer can pre-empt the auction by paying a posted price that is selected by the seller.[7] During the time when our data were collected, eBay's website did not reveal whether or not an auction ended in a transaction. We assume that all auctions that attracted a bid ended with a sale, and that the winner was the buyer with the highest standing bid.

The sale prices for TI-83 calculators in retail stores in the summer of 2003 were between \$80 and \$100 for new items and between \$40 and \$60 for used ones. The final prices of the auctions we have in our dataset differ substantially—some items are new, some are used, and some come with additions like cases or operating manuals. Our dataset also contains auctions offering these additions without the calculator, or more than one calculator bundled together. Unfortunately we do not observe product characteristics, and thus we are unable to eliminate the heterogeneity caused by the variation in items. In order to attain greater consistency among the items in our dataset we keep the auctions that have a transaction price between \$20 and \$110.

There are 1,817 unique auctions completed in the 6-week period. We observe 4,404 bidders who submit a total of 13,240 maximum bids.[8] Moreover, we observe 192 unique auctions that did not receive any bids or were cancelled. We exclude those auctions. We collect the following information for each TI-83 auction: auction number, starting bid, number of bids, auction length, winning bid, minimum bid, and number of bidders.

[7] The Buy-It-Now option in such an auction disappears as soon as a maximum bid above the reserve price is submitted, and the auction becomes indistinguishable from one that never had the Buy-It-Now option. Therefore, our dataset likely includes some wiped-out Buy-It-Now auctions.

[8] We detected 229 bids without any bidder ID. Fortunately, most of these bids do not affect the outcomes of the particular auctions.

Table 1 Definitions of variables

Variables	Explanation
Auction number	ID of the auction
Winning bid	Maximum recorded bid of each auction
Starting bid	Starting price of the auction that is set by the seller
Minimum bid	Minimum recorded bid of each auction
Auction length (h)	Length of the auction measured in hours
Number of bids	Total number of bids at the end of the auction
Number of bidders	Total number of bidders at the end of the auction
I	Total number of unsuccessful bidders in previous 10 auctions
Closing interval	Time period during which an auction is next-to-close
Elapsed time	Total time of closing intervals for previous 10 auctions

Table 2 Descriptive statistics for TI-83 calculators

Variable	No. of obs.	Mean	Median	SD	Min	Max
Winning bid	1,817	58.21	53.09	17.7	20	107.52
Starting bid	1,817	15.07	9.99	15.96	0.01	99.99
Minimum bid	1,817	18.74	12	16.65	0.01	99.99
Auction length (h)	1,817	135.3	168	46.25	72	240
Number of bids	1,817	13.7	13	7.04	1	54
Number of bidders	1,817	7.29	7	3.02	1	18
I	1,807	52.71	52	11.61	12	95
Elapsed time (min)	1,806	333.05	230.5	255.25	15	1,916

Table 1 presents a brief description of these variables, and Table 2 provides descriptive statistics. The winning bid is the maximum bid of the second-highest bidder plus the bid increment. The calculators sell for a bit more than \$58 on average, with a standard deviation around \$17. The average starting bid is just above \$15, and the lowest submitted bid is a little below \$19 on average. The average auction lasts 5.5 days and attracts 14 bids. The average number of bidders is 7, with standard deviation equal to 3. We note that the ratio of standard deviation to mean is significantly higher for the number of bidders than for the sale price; this difference is consistent with our assumption that there is more uncertainty about the number of bidders than about their willingness to pay.

We also use a subset of our data to follow the behavior of bidders. We choose a four-day window—July 5 through July 8, 2003—in the middle of the dataset and identify all the bidders that are active in that window. We then track their outcomes forward and backward in time. By choosing a window in the middle of the range of dates, we can avoid truncation problems and observe bidders' entire histories. The window, consisting of a Saturday through a Tuesday, includes both weekends and weekdays, in order to capture any day-effects. We use this subset to study how buyers react after winning or losing an auction. There are 720 unique bidders who submit at least one

Table 3 Summary statistics for the closing intervals

Variable	No. of obs.	Mean	Median	SD	Min	Max
Closing interval (min)	1,816	33.59	14	64.92	0	714

Table 4 Distribution of closing intervals with various lengths

Minutes in the closing interval	Frequency	Percent (%)	Cumulative (%)
0	112	6.17	6.17
1	132	7.27	13.44
2	79	4.35	17.79
3	84	4.63	22.41
4	69	3.80	26.21
5	62	3.41	29.63
6–10	238	13.1	42.73
11–30	538	29.63	72.36
>30	502	27.64	100

maximum bid in the four-day period. In total, those bidders submit 8,186 bids over the entire six-week sample.

3.2 Analysis

First we look at last-minute bidding. We introduce a new and informative approach of studying bid submission times. Many auctions on eBay have ending times that are fairly close together. If buyers submit their bids in whichever auction ends next, then that closeness may generate the appearance of last-minute bidding without any strategic intent by buyers to delay their bids.

For example, if each auction ends 1 min after the previous auction, and bidders participate only in the next auction to close, then *all* bidding will be in the last minute. To distinguish that effect from true last-minute bidding (where buyers wait until competitors can no longer respond), we arrange the auctions sequentially by their closing times and look at bidding behavior during the interval when an auction is the next to close. We find evidence of both sorts of bidding.

A search for “TI-83 calculator” on eBay yields a list of auctions sorted by closing time, with the next auction to close listed at the top. There is thus a natural tendency for buyers to participate in soonest-ending auction. To see how close together auctions end, we define for each auction in our sample the *closing interval*. The closing interval of an auction is the time period during which that auction is the next to close; that is, the time from when the previous auction closes to when the current auction closes.

In Tables 3 and 4 we present descriptive statistics on the lengths of the closing intervals. On average, the closing times for auctions are 33 min apart, with a minimum of 0 (that is, less than 1 min) and a maximum of almost 12 h.[9] The median length of

[9] Thus, since the minimum auction length is three days, each auction in our sample overlaps with at least one other auction. In other words, there are always multiple active auctions.

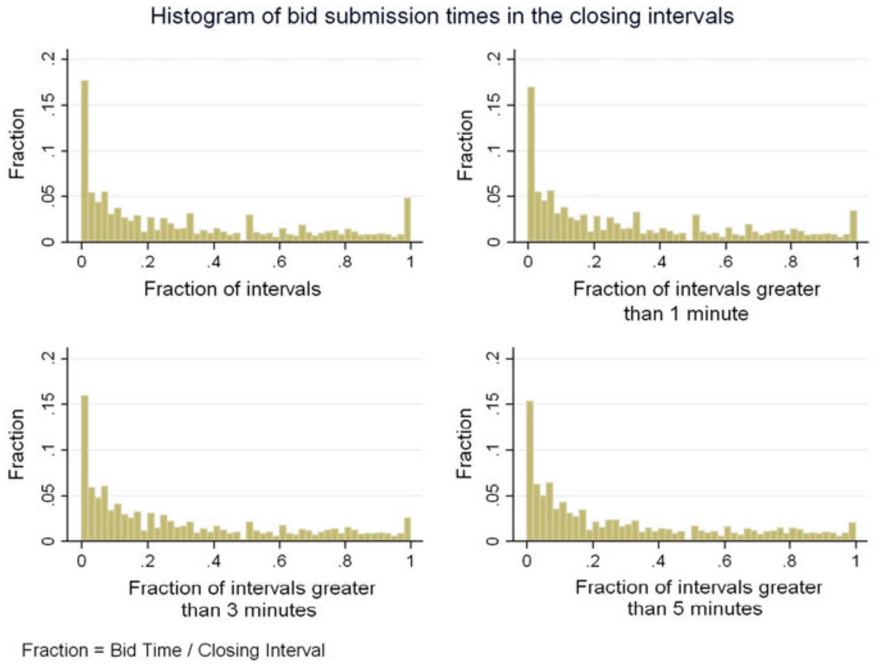

Fig. 1 Bid submission times in the closing intervals

a closing interval is 14 min. Many of the auctions in our dataset close soon after the previous one ends. For more than 6% of the auctions, the closing interval is less than 1 min long. Moreover, almost one third of the auctions have 5 min or less between their closing times.

The prevalence of very short closing intervals may create misleading measures of last-minute bidding. In response, we examine the distribution of bid times relative to the length of the closing interval. The histograms in Fig. 1 show the submission times of maximum bids, measured as the fraction of the auction's closing interval remaining. For example, if an auction's closing interval is 2 min long, then a bid that is submitted 30 s before the end is recorded as being submitted at time 0.25. Each bin in the histograms represents 2% of the length of the closing interval. The first histogram includes all the auctions in our dataset. Subsequent histograms exclude auctions with closing intervals below 1, 3, and 5 min, respectively.

The histograms show a spike at time 1, just when the previous auction ends. Overall, 5% of bids are submitted before 2% of the closing interval has elapsed.[10] That spike becomes smaller as we look at auctions with longer closing intervals. The histograms also show a larger spike in the first bin: the last 2% of the closing interval. In all four cases between 20 and 25% of maximum bids are submitted after 98% of the closing interval has elapsed. Thus, although some late bidding seems to be driven merely from buyers' arriving in the soonest-ending auctions, we also find evidence that some

[10] The first 2% of the median 14-minute closing interval is the first 17 s.

buyers do wait until the very end of auctions to participate: last-minute bidding is not an illusion resulting from closely-spaced auctions.

Figure 2 classifies the histograms of Fig. 1 according to whether or not the bidder has participated in an earlier auction in the dataset. We classify bidders who have submitted a bid in an earlier auction as "incumbents" and all the new participants as "entrants". In all four cases, entrants are more likely to submit a maximum bid in the last 2% of the closing interval than are incumbents. Entrants are also less likely to submit a maximum bid during the first 2% of the interval.

Next, we look at the decisions that are made by bidders after winning or losing an auction. Table 5 classifies the 720 buyers in the sub-sample according to how many auctions they win in the entire six-week period. The table shows that 365 of the bidders win at least one auction, and together those 365 win 508 auctions. Among the 365 winners, 310 (85%) win a single auction. It appears, then, that the assumption of unit demand, which we make in the theoretical section, is reasonable for most bidders.

We also track the 720 bidders from the first auction in which they participate. Figure 3 shows that 135 of the 720 win (W) their first auction. Of those initial winners, a large fraction (110/135) exit (E) eBay's website; the dataset contains no future bids for them. Of the 585 bidders who lose (L) their first auction, most (429) bid (B) in at least one subsequent auction. Of those 429 re-bidders, 366 lose their second auction, and 316 of those 366 bid again. Of the 63 who win on their second try, 53 exit. We conclude that, to a first approximation, buyers have unit demands, and losers of an auction try again in subsequent auctions. We also note that entrants (participating in their first auction) win 22% more frequently than do incumbents: 135/720 versus 63/429.

Finally, to explore how the presence of incumbents affects bidding behavior, for each auction in our sample (except the first ten) we construct a variable I, which is equal to the number of bidders who submitted a maximum bid in at least one of the previous ten auctions, but who did not win an item.[11] The mean number of such incumbents is 52.71. We regress winning bids on I plus day-of-the-week and time-of-day dummy variables in addition to dummy variables for each week to capture any time trend in the data.[12]

Out of concern that the number of incumbents I may be endogenous (if the entry rates of buyers are correlated across time, then I will be correlated with the expected number of new bidders), we instrument for I using the length of the closing interval and the elapsed time of the past ten auctions.[13,14] Elapsed time is likely to be positively

[11] We believe considering the previous ten auctions is reasonable since they correspond to roughly 2 h and beyond that bidders might not be present at their computers. See Table 2 for descriptive statistics.

[12] F tests for the collection of hour, day, and week dummy variables suggest that their explanatory powers for the OLS and the first stage of the 2SLS regressions are statistically significant with associated p values of 0.00. Coefficients on these dummy variables indicate increased bidder participation if an auction closes in the afternoon or evening as well as in the last weeks of the data-collection period. Auctions that end on Fridays, Saturdays, and Sundays attract fewer bidders, as compared to the rest of the week.

[13] To be precise, we define the elapsed time variable for auction n as the time from when auction $n-11$ closes to when auction $n-1$ closes. Table 2 provides descriptive statistics on this variable.

[14] The endogeneity of the number of incumbents, I, might also be due to the lack of information on item characteristics in our dataset. Unless corrected, the omitted explanatory variables might be correlated with the number of incumbents and thus bias our results.

Fig. 2 Bid submission times in the closing intervals, by incumbent status

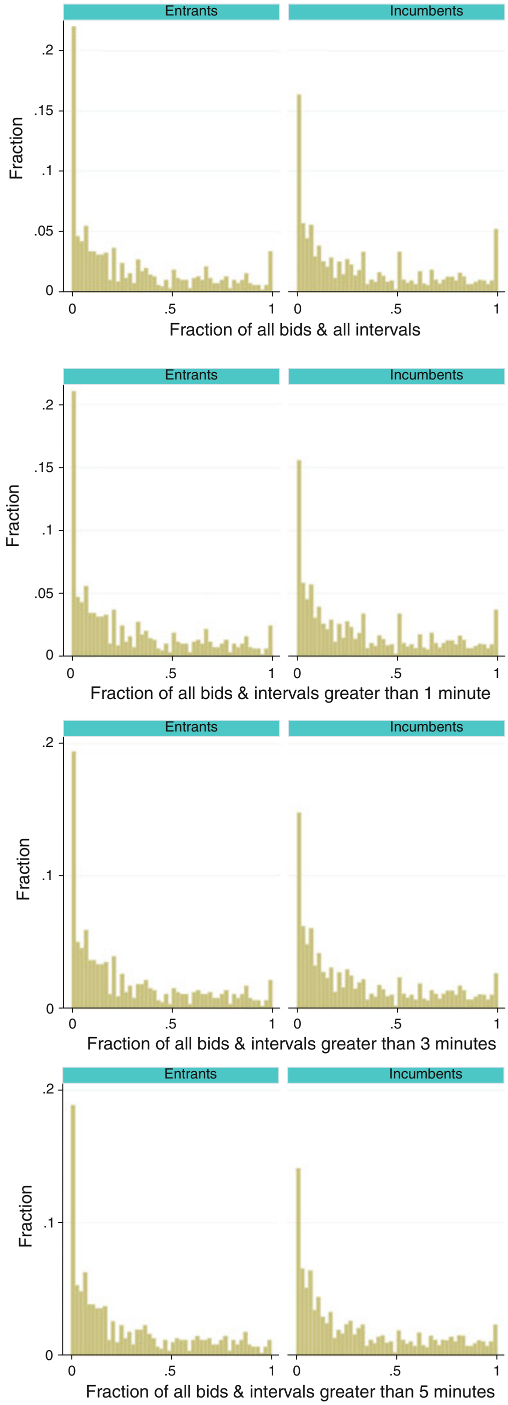

Table 5 The number of bidders and the number of auctions that they won

Number of wins	Number of bidders	Percentage
0	355	49
1	310	43
2	39	8
3	9	
4	2	
5	1	
6	1	
10	1	
15	1	
49	1	

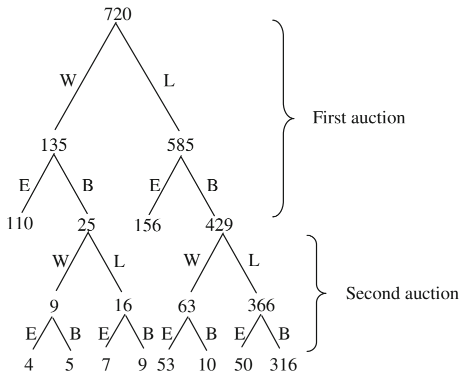

Fig. 3 Bidder behavior in consecutive auctions

correlated with the number of losing bidders since more bidders arrive when the time interval covered by the ten auctions is longer. If the arrival rate of sellers is more or less exogenous, though, elapsed time is likely to be uncorrelated with the number of entrants in the current auction.

The results are presented in Table 6.[15] OLS results indicate no significant effect of the number of incumbents on the winning bid. However, the two-stage least-squares regression yields a positive and statistically significant coefficient: the coefficient is 0.73, with a standard error of 0.2. That is, an additional incumbent increases the expected price in an auction by \$0.73, or 1.25% of the mean price.[16] A log specification yields similar results. Comparison of the exogenous OLS and endogeneity-corrected

[15] The Hansen's J statistic is reported in the table. The Sargan–Hansen test result shows that the instruments are valid.

[16] As a robustness check we calculated the number of incumbents who only bid between the previous 10–20, and 20–30 auctions. We then ran the 2SLS regressions with these variables included. The magnitude and significance of the coefficient for I was stable among various specifications. Furthermore, the newly created variables did not have any significant effect on the winning bid. For brevity these results are not reported here but are available from the corresponding author upon request.

Table 6 Regression of the winning bid on the number of incumbents

Winning bid	OLS	2SLS	
		First-stage regression	IV regression
I	0.0434 (0.0439)	–	0.7306 (0.2008)***
Closing interval	–	0.0113 (0.0041)***	–
Elapsed time	–	0.0116 (0.0013)***	–
Constant	50.7114 (7.1814)***	32.6849 (2.3279)***	24.756 (9.6665)***
R-squared	0.0765	0.4043	–
Hansen's J	–	–	1.090
Hansen's $P > \mathrm{J}$	–	–	0.2965
Number of observations: 1,806			

Figures in parentheses are robust standard errors. Hour, day, and week dummy variables were also included as regressors. Collective F tests for these variables suggest that their explanatory powers are statistically significant for the OLS and the first stage of the 2SLS regressions

*** Significance at 1% level, ** significance at 5% level, * significance at 10% level

2SLS results suggests that the omitted variables are biasing the OLS results negatively. As a result, we conclude that auction prices are higher when the stock of bidders who lost recent auctions is greater.

4 Model

We study the following simplified version of the eBay auction setting.

4.1 Environment

There is an infinite sequence of auctions; each auction involves a single unit of a homogeneous good. All buyers are risk-neutral; their utility is quasilinear in wealth; and they discount the future at a rate $\delta \in (0, 1)$ per period.[17] Each period, up to two new buyers enter, with independent probability p. A buyer's arrival is not observed by others until he participates in an auction. A buyer exits only after he has obtained a unit of the good. We discuss the implications of this assumption and the consequences of relaxing it in the next section. Each buyer wants one unit of the good, and values that one unit at $V > 0$. All buyers are expected-utility maximizers. The structure is common knowledge.

We abstract from the details of eBay's proxy-bidding program and assume continuous-time, second-price, sealed-bid auctions. Auction t runs from time t to $t + 1$, for $t \in \{1, 2, \ldots\}$. The assumption that auctions do not overlap is not important;

[17] The terms "per period" and "per auction" are used interchangeably in this section.

what matters is that they have different ending times.[18] The (publicly observed) reserve price in each auction is the same and is normalized to zero.[19]

A buyer who participates in auction t chooses a time between t and $t+1$ to enter a bid b; all buyers observe when and by whom a bid is submitted. Having entered a bid, a buyer can neither retract nor lower it; he can submit a higher bid at any time before the auction ends. At the end, the item is awarded to the highest bidder at a price that is equal to the second-highest bid. (Ties are broken randomly with equal probabilities). Each buyer observes the history of outcomes and bidders' identities in all auctions that have occurred prior to the current auction.

A (pure) strategy tells a buyer whether, when, and at what amount to bid, as a function of time, history, and his own previous actions. A buyer cannot bid in an auction before that auction begins, or before he enters.

4.2 Outcomes

We construct a symmetric, sequential equilibrium in pure strategies: A buyer who enters in period t bids at the end of auction t, so that the number of incumbents is common knowledge. After entering, a buyer participates in every auction until he wins an item. (We note that this feature of the equilibrium is a simplification relative to the pattern observed in the data. Although, as demonstrated in the previous section, losing bidders in our dataset usually participate again in subsequent auctions, they typically do not bid in the next auction to close). As a preliminary, we define the function $S(n),\ n \in \{1, 2, \ldots\}$, recursively as

$$S(1) = \frac{a}{1-cd} V \text{ and } S(n) = d^{n-1} S(1),$$

where

$$a = \frac{(1-p)^2\,\delta}{1-2p\,(1-p)\,\delta},\quad c = \frac{p^2\delta}{1-2p\,(1-p)\,\delta},\quad \text{and } d = \frac{1-\sqrt{1-4ac}}{2c}.$$

In the equilibrium to be constructed, $S(n)$ is the expected (undiscounted) surplus for an incumbent at the beginning (i.e., before the number of new entrants is known) of a period in which the total number of incumbents is n. That surplus is generated by the following bidding functions $b^i(n)$ and $b^e(n)$ for incumbents and entrants, respectively:

$$b^i(1) = 0,\ b^i(n) = V - \delta S(n-1) \text{ for } n \geq 2, \text{ and } b^e(n) = b^i(n+2).$$

A buyer who obtains an item at price $b^i(n)$ gets surplus $\delta S(n-1)$. $S(n)$ can be interpreted as the expected payoff that the incumbent would get from waiting until the

[18] In the equilibrium that we construct, each active bidder participates in each auction. Papers that examine the selection of buyers across different sellers include Bajari and Hortacsu (2003), Dholakia and Soltysinski (2001), McAfee (1993), Peters and Severinov (2006) and Stryszowska (2004).

[19] Actual eBay auctions have both a public "starting bid" and a hidden "reserve price", as described in Sect. 2.

first auction in which (i) there are no other incumbents left, and (ii) no new buyers enter; and then winning that auction, uncontested, for price zero. As the number of incumbents n rises, the expected wait until that first uncontested auction increases. Thus, $S(n)$ is decreasing (exponentially) in n.

Note that the parameter d is strictly between 0 and 1, and that its limit as the discount factor δ approaches 1 is 1. The same is true of the expression $a/(1-cd)$. For any fixed n, as buyers become more patient, the expected discounting of the time before the first uncontested auction becomes negligible, and the expected surplus $S(n)$ approaches V. That is,

$$\lim_{\delta \to 1} S(n) = V \text{ for } n \in \{1, 2, \ldots\}.$$

For any fixed discount rate δ less than 1, $\lim_{n\to\infty} S(n) = 0$. Thus, in thick markets, bids are very close to V.

Proposition 1 *In auction t, let n_t be the number of buyers who have (i) bid in a previous auction, and (ii) not obtained an item. Let j_t be the number who (i) bid before the end of auction t, and (ii) have not bid in a previous auction. The following strategies and beliefs constitute a sequential equilibrium.*

At time $t+1$, the end of auction t, an incumbent submits a bid of $b^i\,(n_t + j_t)$ if he has bid previously, and a bid of $b^e\,(n_t + j_t + 1)$ otherwise. At the same time, an entrant bids $b^e(n_t + j_t)$, unless he has already submitted a bid earlier in the auction, in which case he bids $b^i\,(n_t + j_t)$.

Bayes' rule gives beliefs (about the number of buyers) at histories reachable in equilibrium. Off-equilibrium, a buyer is believed to be present from his first bid until he wins an item. A buyer who submits his first bid (in any auction) during, but before the end of, an auction is believed to be an entrant from a previous auction. (A history in which an entrant deviates by not bidding is indistinguishable from a history in which that entrant did not arrive, and thus is reachable in equilibrium).

Proof First, we show that these strategies deliver surplus $S(n)$ to an incumbent. Suppose that $n = 1$. With probability $p_0 \equiv (1-p)^2$, there are no entrants, and the single incumbent wins at price zero. With one entrant (probability $p_1 \equiv 2p\,(1-p)$), the incumbent loses and continues to the next auction, in which n is still 1. Finally, with two entrants (probability $p_2 \equiv p^2$), the incumbent and one entrant lose, and n increases to 2. Thus, S must satisfy

$$\begin{aligned} &S(1) = p_0 V + p_1 \delta S(1) + p_2 \delta S(2) \\ &\Leftrightarrow \\ &S(1) = \frac{p_0}{1 - p_1\delta} V + \frac{p_2\delta}{1 - p_1\delta} S(2). \end{aligned} \tag{1}$$

Similarly, with $n > 1$ incumbents, S must satisfy

$$S(n) = p_0 \delta S(n-1) + p_1 \delta S(n) + p_2 \delta S(n+1). \tag{2}$$

The second-order difference equation described by Expressions 1 and 2 has a solution of the form $f(n) = (k_1)^n k_2$. Straightforward algebra then yields the function $S(n)$ described above as the solution.

Next, we show that the strategies are best responses. Consider an incumbent who has bid previously, and let n be the number of incumbents (including those who bid for the first time during the current auction). He is willing to bid at any time, including $t+1$, since his presence is already known.[20] Any bid up to $b^e(n)$ (or no bid) yields expected continuation $S(n)$. Bidding above $b^e(n)$ wins for sure, at price $b^i(n)$ if there is no entrant and at price $b^e(n)$ otherwise, yielding

$$p_0 \delta S(n-1) + (p_1 + p_2)\,\delta S(n+1). \tag{3}$$

Since $S(\cdot)$ is decreasing, $S(n)$ exceeds Expression 3. Finally, bidding $b^e(n)$ wins at price $b^i(n)$ if no entry occurs. If $m \in \{1, 2\}$ buyers enter, then the incumbent wins at price $b^e(n)$ with probability $\frac{1}{m+1}$, and otherwise proceeds to the next auction with $n+m-1$ other incumbents. Thus, bidding $b^e(n)$ yields

$$p_0 \delta S(n-1) + p_1 \left(\frac{1}{2} \delta S(n) + \frac{1}{2} \delta S(n+1) \right) + p_2 \delta S(n+1) < S(n).$$

A similar argument establishes that bidding $b^i(n)$ at the end of the auction is optimal for an entrant who (off equilibrium) has placed a bid earlier in the auction.

Now consider an entrant when $n > 0$. Since bidding before the end causes other buyers to bid higher, he prefers to bid at the last minute (if at all). If he bids, $b^e(n)$ is optimal. Call $S^e(n)$ the expected payoff from bidding $b^e(n)$ (or higher):

$$S^e(n) = (1-p)\delta S(n-1) + p\delta S(n+1).$$

If another buyer enters (probability p), the entrant gets surplus $\delta S(n+1)$ either from winning the current auction at price $b^e(n)$ or proceeding to the next auction with n other incumbents. With no second entrant, the entrant wins at price $b^i(n)$. A bid strictly between $b^i(n)$ and $b^e(n)$ also yields $S^e(n)$. Bidding less than $b^i(n)$ loses, and thus yields

$$(1-p)\delta S(n) + p\delta S(n+1) < S^e(n).$$

Bidding $b^i(n)$ yields

$$(1-p)\left(\frac{1}{n+1} \delta S(n-1) + \frac{n}{n+1} \delta S(n) \right) + p\delta S(n+1) < S^e(n).$$

When $n = 0$, $b^e(0)$ is optimal, since the payoff from any positive bid is

$$S^e(0) = (1-p)V + p\delta S(1).$$

20 Introducing a small probability that incumbents exit exogenously between auctions would make incumbents strictly prefer to bid at the last minute, as we discuss in the next section.

Thus, an entrant's choice is between bidding $b^e(n)$ or "hiding" by not bidding. Let $S^h(n)$ denote the payoff to a "hidden" incumbent (one who arrived in a previous auction but did not bid) from submitting the equilibrium bid $b^e(n+1)$, when there are n other incumbents. (We show below that $b^e(n+1)$ is optimal). That bid ensures that the hidden incumbent will win, at price $b^i(n)$ if there is no entrant and at $b^e(n)$ otherwise. Thus, the value of $S^h(n)$ is

$$\begin{aligned} &S^h(0) = p_0 V + (p_1 + p_2)\,\delta S(1), \text{ and} \\ &S^h(n) = p_0 \delta S(n-1) + (p_1 + p_2)\,\delta S(n+1) \text{ for } n \geq 1. \end{aligned} \tag{4}$$

$S^e(n_t)$ exceeds $E\left[S^h(n_{t+1})\,|n_t\right]$ (the proof is in the appendix), so an entrant's best response is to bid $b^e(n)$ rather than "hide":

Lemma 1 $S^e(n_t) > S^h(n_t) \geq \delta E[S^h(n_{t+1})|n_t]$ *for all* $n_t \geq 0$.

It remains only to demonstrate that bidding $b^e(n+1)$ is optimal for a "hidden" incumbent. First, we show that if he bids, $b^e(n+1)$ is optimal. (Like an entrant, he prefers to bid at the last minute). Any bid higher than $b^e(n)$ yields payoff $S^h(n)$. Suppose that $n \geq 1$. (If $n = 0$, the argument is similar). Then bidding $b^e(n)$ yields

$$p_0 \delta S(n-1) + p_1 \delta S(n+1) + p_2 \left(\frac{1}{3}\delta S(n+1) + \frac{2}{3}\delta S(n+2)\right) < S^h(n),$$

and bidding strictly between $b^i(n)$ and $b^e(n)$ yields

$$p_0 \delta S(n-1) + p_1 \delta S(n+1) + p_2 \delta S(n+2) < S^h(n).$$

Bidding $b^i(n)$ yields

$$p_0 \left(\frac{1}{n+1}\delta S(n-1) + \frac{1}{n+1}\delta S(n)\right) + p_1 \delta S(n+1) + p_2 \delta S(n+2) < S^h(n),$$

and bidding below $b^i(n)$ yields $S(n+1) < S^h(n)$. Thus, $b^e(n+1)$ is optimal.

To see that a hidden incumbent does better by bidding $b^e(n+1)$ than by waiting, note that Lemma 1 shows that

$$S^h(n_t) \geq \delta E\left[S^h(n_{t+1})\,|n_t\right].$$

The law of iterated expectations, together with the fact that a buyer must eventually bid to make any surplus, then implies that bidding $b^e(n+1)$ is optimal. □

5 Concluding Remarks

We have presented a model of sequential auctions of identical goods with stochastic entry in which losing bidders bid again. The equilibrium is fully revealing: Entrants reveal their presence in the same period that they arrive.

In general, full revelation is difficult to obtain in sequential auctions because of the information leakage problem. A bidder may be reluctant to submit a high bid in auction t, because if he loses his competitors will deduce that he has a high valuation and thus bid more aggressively in auction $t + 1$. Our infinite-horizon model focuses on bidders' learning about the number of rivals rather than about their valuations. In equilibrium, the bidding advantage given to an entrant outweighs the future benefit from convincing competitors that he is absent, and thus he is willing to reveal himself.

The equilibrium has several features of empirical interest. First, bids depend upon the state of competition. When the number of buyers is high, the chances of winning a unit at favorable prices fall, and bidders bid more aggressively. This result may seem obvious, but in many of the theoretical and empirical models of eBay auctions, bidders are assumed to bid their value independently of the number of rivals. Second, entrants in an auction are more likely to win than are incumbents. This selection effect arises from the fact that the entrant is better informed than are incumbents about the number of competitors, and not because of differences in valuations. Third, entrants bid at the last minute.

A key characteristic of the equilibrium is that an incumbent's bid is set at the level such that winning yields the same surplus as the expected surplus from waiting until all of the other incumbents are "cleared out" and then winning the next auction with no competition. The incumbents' bids (and prices) are thus increasing in the total number of incumbents, so that entrants prefer to conceal their presence until the very end of the auction, when it is too late for the incumbents to react. The last-minute bidding implies that bidders learn whether they are losers only at the end of an auction, at which point, they bid again in a subsequent auction.

Incumbents bid in every auction until they win, and they are indifferent as to when to bid during an auction. These results follow from the assumption that bidders do not exit except by winning. If we introduce a probability of exogenous exit, then incumbents would have private information about their participation in much the same way that entrants have. The private information would give incumbents an incentive not to reveal themselves until the last minute. In fact, incumbents would have an incentive to pretend to exit, lowering their rivals' expectations of the number of bidders and bids in subsequent auctions, and then bid in the next auction.

We conjecture that if we add a very small probability of exit, then the new equilibrium will involve a very small probability of an incumbent's hiding in each auction conditional on not exiting. In other words, incumbents will play a mixed strategy in which they are indifferent between bidding at the last minute in the current auction and waiting until the next auction to bid. The intuition is that the rivals' belief that an incumbent has exited after not bidding would have to be very low in order to balance the gain from hiding (in the form of slightly lower competitors' bids in the future) and the cost of missing a chance to win the product for a low price in the current auction (if other incumbents choose to hide).[21]

[21] This kind of "low probability of behavioral type implies low probability of mimicking" result is standard in reputation models. See, for example, Mailath and Samuelson (2006, ch. 17).

Consequently, our equilibrium may be robust in the following sense: If there is a 1% chance of exit, then incumbents bid with 99% probability instead of 100% and are certain to bid at the last minute. Confirming this conjecture is beyond the scope of this paper.[22] More generally, allowing bidders to exit would be an interesting extension of our model. A preliminary analysis of the case where bidders become "discouraged" and exit when the number of incumbents exceeds some limit $\overline{n}$ yields similar results to our baseline model.

Another interesting way to generalize our model is to allow heterogeneity in bidders' valuations. In that setting, our baseline model's extreme prediction that incumbents win only when no entrants are present would no longer hold. Instead, we speculate that we would obtain a result showing that an incumbent bidder has a lower probability of winning any given auction than does an entrant with the same valuation.

We note also that the model's prediction that bids are increasing in the number of incumbents n might fail to hold in a common-value setting with uncertainty (because of the winner's curse effect), or if n increases endogenously due to an increase in supply.

As Roth and Ockenfels (2002) show, last-minute bidding behavior is sensitive to the details of auction closing rules. In our model, as in Roth and Ockenfels, last-minute bidding depends on the hard closing rule. With a soft close (meaning that the auction is extended whenever a maximum bid is submitted near the closing time), rivals always have a chance to respond to the information that is revealed by a late bid, and entrants cannot effectively conceal themselves. Another interesting topic for future research is to try to extend our model to correspond to the rules of other online auction sites, such as http://Amazon.com or http://ubid.com.

Appendix

Proof of Lemma 1 Because $p_0 = (1-p)^2 < (1-p)$ and $S(\cdot)$ is decreasing, $S^e(n) > S^h(n)$. We finish by showing that

$$S^h(n_t) \geq \delta E\left[S^h(n_{t+1})\,|n_t\right] \text{ for all } n_t \geq 0:$$

If $n_t \geq 2$, then

$$\begin{aligned}
\delta E\left[S^h(n_{t+1})\,|n_t\right] &= p_0\delta S^h(n_t-1) + p_1\delta S^h(n_t) + p_2\delta S^h(n_t+1) \\
&= \begin{cases} (p_0\delta)^2 S(n_t-2) + p_0(p_1+p_2)\delta^2 S(n_t) \\ +p_0p_1\delta^2 S(n_t-1) + p_1(p_1+p_2)\delta^2 S(n_t+1) \\ +p_0p_2\delta^2 S(n_t) + p_2(p_1+p_2)\delta^2 S(n_t+2) \end{cases} \\
&= \begin{cases} p_0\delta\left[p_0\delta S(n_t-2) + p_1\delta S(n_t-1) + p_2\delta S(n_t)\right] \\ +p_1\delta\left[p_0\delta S(n_t) + p_1\delta S(n_t+1) + p_2\delta S(n_t+2)\right] \\ +p_2\delta\left[p_0\delta S(n_t) + p_1\delta S(n_t+1) + p_2\delta S(n_t+2)\right] \end{cases} \\
&= p_0\delta S(n_t-1) + (p_1+p_2)\delta S(n_t+1) \\
&= S^h(n_t).
\end{aligned}$$

The cases $n_t \in \{0, 1\}$ are similar. □

[22] One technical complication is that the tractable second-order difference equation in Expressions 1 and 2 would become of order $n+2$: Each of two entrants might arrive, and each of n incumbents might exit.

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